Some New Analytic Mean Graphs

P. Lawrence Rozario Raj[1] and C.Ranjitha[2]
[1] Asst Prof of Mathematics, St. Joseph's College (Autonomous), Tiruchirappalli
[2]M.Sc. Mathematics, St. Joseph's College (Autonomous), Tiruchirappalli

1. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges, for terms not defined here, we refer to Harary [2]. For standard terminology and notations related to graph labeling, we refer to Gallian [1]. In [4], Tharmaraj *et al*. introduce the concept of an analytic mean labeling of graph. Analytic mean labeling of various types of graphs are presented in [3,5]. The brief summaries of definition which are necessary for the present investigation are provided below.

2. Definitions

Definition 2.1

A graph labeling is the assignment of unique identifiers to the edges and vertices of a graph.

Definition 2.2 [6]

The shadow graph $D_2(G)$ of a connected graph G is obtained by taking two copies of G say G′ and G″. Join each vertex u′ in G′ to the neighbors of the corresponding vertex u″ in G″.

Definition 2.3 [4]

A (p,q) graph G(V,E) is said to be an analytic mean graph if it is possible to label the vertices v in V with distinct from 0,1,2,…, p−1 in such a way that when each edge e = uv is labeled with $f^*(e = uv) = \frac{|[f(u)]^2 - [f(v)]^2|}{2}$

if $|[f(u)]^2 - [f(v)]^2|$ is even and

$$\frac{|[f(u)]^2 - [f(v)]^2| + 1}{2}$$

if $|[f(u)]^2 - [f(v)]^2|$ is odd and the edge labels are distinct. In this case, f is called an analytic mean labeling of G. A graph with an analytic mean labeling is called an analytic mean graph.

3. Main Results

Theorem : 3.1

Let G be any analytic mean graph of order m (≥ 3) and size q, and $K_{2,n}$ be a bipartite graph with the bipartition $V = V_1 \cup V_2$ with $V_1 = \{w_1, w_2\}$ and $V_2 =$

$\{u_1,u_2,...,u_n\}$. Then the graph $G * K_{2,n}$ obtained by identifying the vertices w_1 and w_2 of $K_{2,n}$ with that labeled 0 and labeled 2 respectively in G is also analytic mean graph.

Proof

Let G be a graph of order m and size q.

Let $v_1,v_2,...,v_m$ and $e_1,e_2,...,e_q$ be the vertices and edges of G.

Let G be any analytic mean graph with mean labeling f.

Then the induced edge labels of G are distinct and lies between

1 to $\frac{(m-1)^2}{2}$ (or) $\frac{(m-1)^2+1}{2}$.

Let v_j and v_k be the vertices having the labels 0 and 2 in G.

Let $V = V_1 \cup V_2$ be the bipartition of $K_{2,n}$ such that

$V_1 = \{w_1,w_2\}$ and $V_2 = \{u_1, u_2,..., u_n\}$.

Now identify the vertices w_1 and w_2 of $K_{2,n}$ with that labeled 0 and labeled 2 respectively in G.

Define $h : V(G) \to \{0,1,2,\ldots, m+n-1\}$ by $h(v_i) = f(v_i)$ for $1 \le i \le m$

$h(u_i) = m + i-1$ for $1 \le i \le n$

Let h^* be the induced edge labeling of h. Then $h^*(e_i) = f^*(e_i)$ for $1 \le i \le q$

For $1 \le i \le n$

$$h^*(v_ju_i) = \begin{cases} \frac{(m+i-1)^2+1}{2} & \text{if } m+i-1 \text{ is odd} \\ \frac{(m+i-1)^2}{2} & \text{if } m+i-1 \text{ is even} \end{cases}$$

$$h^*(v_ku_i) = \begin{cases} \frac{(m+i-1)^2-3}{2} & \text{if } m+i-1 \text{ is odd} \\ \frac{(m+i-1)^2-4}{2} & \text{if } m+i-1 \text{ is even} \end{cases}$$

Then the induced edge labels $K_{2,n}$ are distinct and lies between

$$\frac{m^2-4}{2} \text{ (or) } \frac{m^2-3}{2} \text{ and } \frac{(m+n-1)^2}{2} \text{ (or) } \frac{(m+n-1)^2+1}{2}.$$

Also, the induced edge labels of G are distinct and lies between

$$1 \text{ and } \frac{(m-1)^2}{2} \text{ (or) } \frac{(m-1)^2+1}{2}.$$

Then the induced edge labels of $G*K_{2,n}$ are distinct. Hence $G*K_{2,n}$ is analytic mean graph.

Example 3.1

Analytic mean labeling of G and $G*K_{2,5}$ are given in figure 3.1.

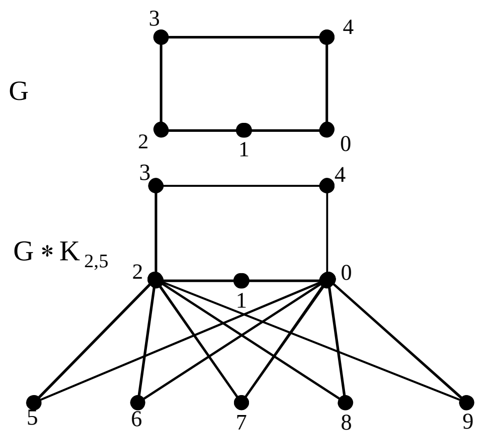

Figure 3.1

Theorem 3.2

Let G be any analytic mean graph of order m (≥ 4) and size q, and $K_{3,n}$ be a bipartite graph with the bipartition

$V = V_1 \cup V_2$ with $V_1 = \{w_1, w_2, w_3\}$

and $V_2 = \{u_1, u_2,..., u_n\}$.

Then the graph $G*K_{3,n}$ obtained by identifying the vertices w_1, w_2 and w_3 of $K_{3,n}$ with that labeled 0, labeled 2 and labeled 3 respectively in G is also analytic mean graph.

Proof

Let G be a graph of order m and size q.

Let $v_1,v_2,...,v_m$ and $e_1,e_2,...,e_q$ be the vertices and edges of G.

Let G be any analytic mean graph with mean labeling f.

Then the induced edge labels of G are distinct and lies between

$$1 \text{ to } \frac{(m-1)^2}{2} \text{ (or) } \frac{(m-1)^2+1}{2}.$$

Let v_j, v_k and v_r be the vertices having the labels 0,2 and 3 in G.

Let $V = V_1 \cup V_2$ be the bipartition of $K_{3,n}$ such that $V_1 = \{w_1, w_2, w_3\}$ and $V_2 = \{u_1, u_2, \ldots, u_n\}$. Now identify the vertices w_1, w_2 and w_3 of $K_{3,n}$ with that labeled 0, labeled 2 and labeled 3 respectively in G.

Define $h : V(G) \rightarrow \{0,1,2,\ldots, m+n-1\}$ by

$h(v_i) = f(v_i)$ for $1 \leq i \leq m$

$h(u_i) = m + i - 1$ for $1 \leq i \leq n$

Let h^* be the induced edge labeling of h.

Then $h^*(e_i) = f^*(e_i)$ for $1 \leq i \leq q$

For $1 \leq i \leq n$

$$h^*(v_j u_i) = \begin{cases} \dfrac{(m+i-1)^2+1}{2} & \text{if } m+i-1 \text{ is odd} \\ \dfrac{(m+i-1)^2}{2} & \text{if } m+i-1 \text{ is even} \end{cases}$$

$$h^*(v_k u_i) = \begin{cases} \dfrac{(m+i-1)^2-3}{2} & \text{if } m+i-1 \text{ is odd} \\ \dfrac{(m+i-1)^2-4}{2} & \text{if } m+i-1 \text{ is even} \end{cases}$$

$$h^*(v_r u_i) = \begin{cases} \dfrac{(m+i-1)^2-9}{2} & \text{if } m+i-1 \text{ is odd} \\ \dfrac{(m+i-1)^2-8}{2} & \text{if } m+i-1 \text{ is even} \end{cases}$$

Then the induced edge labels $K_{3,n}$ are distinct and lies between $\dfrac{m^2-9}{2}$ (or) $\dfrac{m^2-8}{2}$ and $\dfrac{(m+n-1)^2}{2}$ (or) $\dfrac{(m+n-1)^2+1}{2}$.

Also, the induced edge labels of G are distinct and lies between

1 and $\dfrac{(m-1)^2}{2}$ (or) $\dfrac{(m-1)^2+1}{2}$.

Then the induced edge labels of $G * K_{3,n}$ are distinct.

Hence $G * K_{3,n}$ is analytic mean graph.

Example 3.2

Analytic mean labeling of G and $G * K_{3,4}$ are given in figure 3.2.

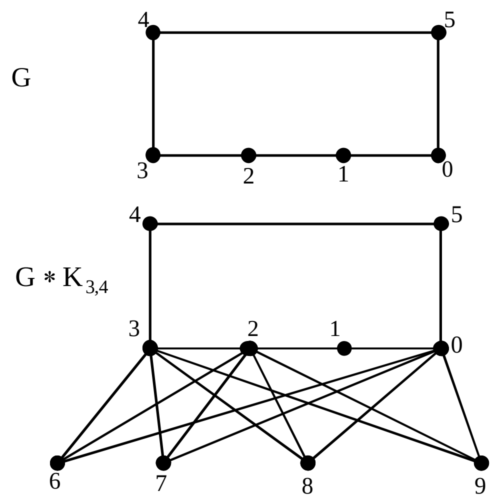

Figure 3.2

Theorem 3.3

$D_2(K_{1,n})$ is an analytic mean graph.

Proof

Let v, v_1, v_2, …, v_n be the vertices of the first copy of $K_{1,n}$ and $v', v'_1, v'_2, \ldots v'_n$ be the vertices of the second copy of $K_{1,n}$ where v and v' are the respective apex vertices.

Let G be $D_2(K_{1,n})$.

Then $|V(G)| = 2n + 2$ and $|E(G)| = 4n$.

Define $f : V(G) \to \{0,1,2,\ldots, 2n+1\}$ by

$f(v) = 0$,

$f(v') = 2$,

$f(v_i) = 2i-1$, for $1 \leq i \leq n$

$f(v'_n) = 2n+1$,

$f(v'_i) = 2i+2$, for $1 \leq i \leq n-1$

Let f^* be the induced edge labeling of f. Then

$$f^*(vv_i) = \frac{(2i+1)^2+1}{2}, \text{ for } 1 \le i \le n$$

$$f^*(v\,v'_i) = \frac{(2i+2)^2}{2}, \text{ for } 1 \le i \le n-1$$

$$f^*(v\,v'_n) = \frac{(2n+1)^2+1}{2}$$

$$f^*(v'\,v_i) = \frac{(2i+1)^2-3}{2}, \text{ for } 1 \le i \le n$$

$$f^*(v'\,v'_i) = \frac{(2i+2)^2-4}{2}, \text{ for } 1 \le i \le n-1$$

$$f^*(v'\,v'_n) = \frac{(2n+1)^2-3}{2}$$

Then the induced edge labels are $\{1,2,3,\ldots, \frac{(2n+1)^2+1}{2}\}$.

Therefore, $D_2(K_{1,n})$ is an analytic mean graph.

Example 3.3

Analytic mean labeling of $D_2(K_{1,4})$ is given in figure 3.3.

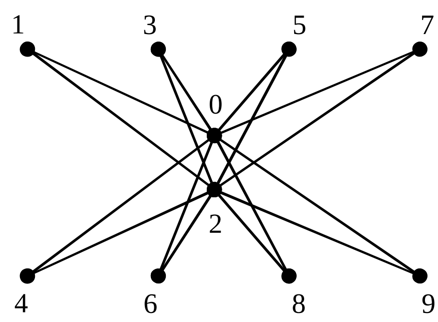

Figure 3.3

4. Conclusion

In this paper, an analytic mean labeling of $G * K_{2,n}$, $G * K_{3,n}$ and $D_2(K_{1,n})$ are presented.

References

1. J. A. Gallian, A dynamic Survey of Graph Labeling, *The Electronic Journal of Combinatorics*, 16, # DS6, 2015.
2. F. Harary, Graph Theory, Addison-Wesley, Reading, Mass, 1972.
3. P. Lawrence Rozario Raj and K. Vivek, Analytic Mean Labeling of Cycle Related Graphs, *International Journal of Innovative Science, Engineering & Technology*, Vol.2, No. 8, 2015, pp.897-901.
4. T. Tharmaraj and P.B.Sarasija, Analytic Mean Graphs, *Int. Journal. of Math. Analysis*, Vol 8, No.12, 2014, pp.595-609.
5. T. Tharmaraj and P.B. Sarasija, Analytic Mean Labelled Graphs, *International Journal of Mathematical Archive*, Vol 5, No. 6, 2014, pp. 136-146.
6. S.K.Vaidya and Lekha Bijukumar, New Mean Graphs, *International J. Math. Combin.* Vol.3, 2011, pp. 107-113.